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Mesons of the f_0 family in processes $\pi\pi \to \pi\pi, K\overline{K}$ up to 2 GeV and the chiral-symmetry breaking

Yu.S. Surovtsev[1,a], D. Krupa[2,b], and M. Nagy[2,c]

[1] Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Dubna 141 980, Moscow Region, Russia
[2] Institute of Physics, Slovak Academy of Sciences, Dúbravská cesta 9, 842 28 Bratislava, Slovakia

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Abstract. In a combined analysis of the experimental data on the coupled processes $\pi\pi \to \pi\pi, K\overline{K}$ in the channel with $I^G J^{PC} = 0^+0^{++}$, the various scenarios of these reactions (with different numbers of resonances) are considered. In a model-independent approach, based only on analyticity and unitarity, a resonance is represented by a pole cluster (poles on the Riemann surface) of the definite type that is defined by the state nature. The best scenario contains the resonances $f_0(665)$ (with properties of the σ-meson), $f_0(980)$ (with a dominant $s\bar{s}$ component), $f_0(1500)$ (with a dominant flavour-singlet, *e.g.*, glueball component) and the $f_0(1710)$ (with a considerable $s\bar{s}$ component). If the $f_0(1370)$ exists, it has a dominant $s\bar{s}$ component. The coupling constants of the observed states with the considered channels and the $\pi\pi$ and $K\overline{K}$ scattering lengths are obtained. The conclusion on the linear realization of chiral symmetry is drawn.

PACS. 11.55.Bq Analytic properties of S-matrix – 11.80.Gw Multichannel scattering – 12.39.Mk Glueball and nonstandard multi-quark/gluon states – 14.40.Cs Other mesons with $S = C = 0$, mass < 2.5 GeV

1 Introduction

In the scalar mesonic sector, many states have been discovered at present [1], however, their assignment to quark-model configurations is problematic —one can compare various variants of that assignment, for example, [2–8]. It seems that the problem of scalar mesons is far off the solution up to now. For instance, at present, additional arguments have been added by N.N. Achasov [9] in favour of the 4-quark nature of $f_0(980)$ and $a_0(980)$ mesons on the basis of interpretation of the experimental data on the decays $\phi \to \gamma\pi^0\pi^0, \gamma\pi^0\eta$ [10]. On the other hand, F.E. Close and A. Kirk [11] have shown that mixing between the $f_0(980)$ and $a_0(980)$ radically affects some existing predictions of their production in ϕ radiative decay. Generally, the 4-quark interpretation, beautifully solving the old problem of the unusual properties of scalar mesons, sets new questions. Where are the 2-quark states, their radial excitations and the other members of 4-quark multiplets $9, 9^*, 36$ and 36^*, which are predicted to exist below 2.5 GeV [12]?

The discovered states in the scalar sector and their properties do not allow one to make up the scalar $q\bar{q}$ nonet and to solve other exiting questions up to now. Generally, difficulties in understanding the scalar-isoscalar sector seem to be related to both the hard-accounting influence of the vacuum (and such effects as the instanton contributions) and a strong model dependence of an information about wide multichannel states.

Earlier, we have shown [13] that an inadequate description of multichannel states (to which scalar mesons belong) gives not only their distorted parameters when analyzing data but also can cause the fictitious states when one neglects important (even energetic-closed) channels. Obviously, it is important to have a model-independent information on investigated states and on their QCD nature. It can be obtained only on the basis of the first principles (analyticity, unitarity) immediately applied to experimental data analysis. Earlier, we have proposed this method for 2- and 3-channel resonances and developed the concept of standard clusters (poles on the Riemann surface) as a qualitative characteristic of a state and a sufficient condition of its existence [13]. We outline this below for the 2-channel case of the coupled processes $\pi\pi \to \pi\pi, K\overline{K}$ in the channel with $I^G J^{PC} = 0^+0^{++}$. Since, in this work, a main stress is laid on studying lowest states, it is sufficient

[a] e-mail: surovcev@thsun1.jinr.ru
[b] e-mail: krupa@savba.sk
[c] e-mail: fyzinami@nic.savba.sk

to restrict oneself to a two-channel approach when considering simultaneously the coupled processes $\pi\pi \to \pi\pi, K\overline{K}$, though in the future it is necessary to take into account the thresholds of other coupled processes, first of all, of $\eta\eta$ and $\eta\eta'$ scatterings. In this work, we are going to show that the large background, which one has obtained earlier in various analyses of the s-wave $\pi\pi$ scattering [1], hides, in reality, the σ-meson [14] below 1 GeV and the effect of the left-hand branch point. Therefore, in the uniformizing variable, one must take into account, besides the branch points corresponding to the thresholds of the processes $\pi\pi \to \pi\pi, K\overline{K}$, also the left-hand branch point at $s = 0$, related to the background in which the crossing-channel contributions are contained [15]. Furthermore, we shall obtain definite indications of the QCD nature of other f_0-resonances and of the linear realization of chiral symmetry.

Note that recent new analyses of old and new experimental data found a candidate for the σ-meson below 1 GeV (see, *e.g.*, [2,3,16–20]). However, these analyses use either the Breit-Wigner form that is insufficiently-flexible even if modified or the K-matrix formalism without taking account of the energetic-closed (maybe, important) channels, or specific forms of interactions in the quark models; therefore, one cannot talk about the model independence of results. Besides, in these analyses, a large $\pi\pi$ background is obtained.

The layout of this paper is as follows. In sect. 2, we outline the two-coupled-channel formalism, determine the pole clusters on the Riemann surface as characteristics of multichannel states, and introduce a new uniformizing variable, allowing for the branch points of the right-hand (unitary) and left-hand cuts of the $\pi\pi$ scattering amplitude. In sect. 3, we analyze simultaneously experimental data on the processes $\pi\pi \to \pi\pi, K\overline{K}$ in the isoscalar s-wave on the basis of the presented approach. Note that earlier we have shown in two approaches [13] and [15] without and with taking into account the left-hand branch point $\sqrt{s}$ in the uniformizing variable, respectively, that the minimal scenario of the simultaneous description of two coupled processes $\pi\pi \to \pi\pi, K\overline{K}$ is realized without the $f_0(1370)$ and $f_0(1710)$ that are in the Particle Data Group tables. Therefore, we consider also the variants of description of the indicated processes including these states separately as well as simultaneously, and obtain indications of their QCD nature different from the results of many other works (note that our approach is based on the first principles and is free from dynamic assumptions, therefore, our results are rather model independent). In the conclusion, the obtained results are discussed.

2 Two-coupled-channel formalism

Here we restrict ourselves to a 2-channel consideration of the coupled processes $\pi\pi \to \pi\pi, K\overline{K}$. Therefore, we have the 2-channel S-matrix determined on the 4-sheeted Riemann surface. The matrix elements $S_{\alpha\beta}$, where $\alpha, \beta = 1(\pi\pi), 2(K\overline{K})$, have the right-hand cuts along the real axis of the s-plane, starting at $4m_\pi^2$ and $4m_K^2$, and the left-hand cuts, beginning at $s = 0$ for S_{11} and at $4(m_K^2 - m_\pi^2)$ for S_{22} and S_{12}. The Riemann-surface sheets are numbered according to the signs of analytic continuations of the channel momenta

$$k_1 = (s/4 - m_\pi^2)^{1/2}, \qquad k_2 = (s/4 - m_K^2)^{1/2} \qquad (1)$$

as follows: signs $(\mathrm{Im}k_1, \mathrm{Im}k_2) = ++, -+, --, +-$ correspond to the sheets I, II, III, IV.

To obtain the resonance representation on the Riemann surface, we express analytic continuations of the matrix elements to the unphysical sheets $S_{\alpha\beta}^L$ ($L = \mathrm{II}, \mathrm{III}, \mathrm{IV}$) in terms of those on the physical sheet $S_{\alpha\beta}^{\mathrm{I}}$:

$$\begin{aligned}
&S_{11}^{\mathrm{II}} = \frac{1}{S_{11}^{\mathrm{I}}}, \qquad S_{11}^{\mathrm{III}} = \frac{S_{22}^{\mathrm{I}}}{\det S^{\mathrm{I}}}, \qquad S_{11}^{\mathrm{IV}} = \frac{\det S^{\mathrm{I}}}{S_{22}^{\mathrm{I}}},\\
&S_{22}^{\mathrm{II}} = \frac{\det S^{\mathrm{I}}}{S_{11}^{\mathrm{I}}}, \qquad S_{22}^{\mathrm{III}} = \frac{S_{11}^{\mathrm{I}}}{\det S^{\mathrm{I}}}, \qquad S_{22}^{\mathrm{IV}} = \frac{1}{S_{22}^{\mathrm{I}}}, \qquad (2)\\
&S_{12}^{\mathrm{II}} = \frac{iS_{12}^{\mathrm{I}}}{S_{11}^{\mathrm{I}}}, \qquad S_{12}^{\mathrm{III}} = \frac{-S_{12}^{\mathrm{I}}}{\det S^{\mathrm{I}}}, \qquad S_{12}^{\mathrm{IV}} = \frac{iS_{12}^{\mathrm{I}}}{S_{22}^{\mathrm{I}}},
\end{aligned}$$

Here $\det S^{\mathrm{I}} = S_{11}^{\mathrm{I}}S_{22}^{\mathrm{I}} - (S_{12}^{\mathrm{I}})^2$; $(S_{12}^{\mathrm{I}})^2 = -s^{-1}\sqrt{(s-4m_\pi^2)(s-4m_K^2)}F(s)$; in the limited energy interval, $F(s)$ is proportional to the squared product of the coupling constants of the considered state with channels 1 and 2. These formulas are convenient by that the S-matrix elements on the physical sheet $S_{\alpha\beta}^{\mathrm{I}}$ have, except for the real axis, only zeros corresponding to resonances, at least, around the physical region that is interesting for us. Formulas (2) immediately give the resonance representation by poles and zeros on the 4-sheeted Riemann surface. One must distinguish between three types of 2-channel resonances described by a pair of conjugate zeros on sheet I: a) in S_{11}, b) in S_{22}, c) in each of S_{11} and S_{22}. As seen from eqs. (2), to the resonances of types a) and b), there corresponds a pair of complex conjugate poles on sheet III shifted relative to a pair of poles on sheet II and IV, respectively. For the states of type c), one must consider the corresponding two pairs of conjugate poles on sheet III. A resonance of every type is represented by a pair of complex-conjugate clusters (of poles and zeros on the Riemann surface) of a size typical of strong interactions. The cluster kind is related to the state nature. The resonance coupled relatively more strongly to the $\pi\pi$-channel than to the $K\overline{K}$ one is described by the cluster of type a); in the opposite case, it is represented by the cluster of type b) (say, the state with the dominant $s\bar{s}$ component); the flavour singlet (*e.g.*, glueball) must be represented by the cluster of type c) as a necessary condition, if this state lies above the thresholds of the considered channels.

Furthermore, according to the type of pole clusters, we can distinguish, in a model-independent way, a bound state of colourless particles (*e.g.*, $K\overline{K}$ molecule) and a $q\bar{q}$ bound state [13,21]. Just as in the 1-channel case, the existence of a particle bound state means the presence of a pole on the real axis under the threshold on the physical sheet, so in the 2-channel case, the existence of a

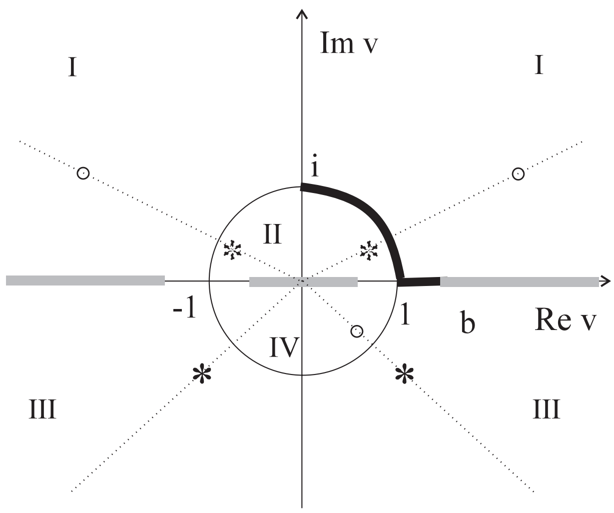

Fig. 1. Uniformization plane for the $\pi\pi$ scattering amplitude.

particle bound state in channel 2 ($K\overline{K}$ molecule) that, however, can decay into channel 1 ($\pi\pi$-decay), would imply the presence of a pair of complex conjugate poles on sheet II under the second-channel threshold without an accompaniment of the corresponding shifted pair of poles on sheet III. Namely, according to this test, earlier, the interpretation of the $f_0(980)$ state as a $K\overline{K}$ molecule has been rejected.

For the simultaneous analysis of experimental data on coupled processes, it is convenient to use the Le Couteur-Newton relations [22] expressing the S-matrix elements of all coupled processes in terms of the Jost matrix determinant $d(k_1, k_2)$, the real analytic function with the only square-root branch points at $k_i = 0$. To take into account, in addition to the latter, also the left-hand branch point at $s = 0$, the uniformizing variable is used [1]

$$v = \frac{m_K\sqrt{s-4m_\pi^2} + m_\pi\sqrt{s-4m_K^2}}{\sqrt{s(m_K^2-m_\pi^2)}}. \tag{3}$$

It maps the 4-sheeted Riemann surface with two unitary cuts and the left-hand cut onto the v-plane. (Note that other authors have used the parameterizations with the Jost functions in analyzing the s-wave $\pi\pi$ scattering in the one-channel approach [24] and in the two-channel one [21]. In latter work, the uniformizing variable k_2 has been used, therefore, their approach cannot be employed near by the $\pi\pi$ threshold.)

In fig. 1, the plane of the uniformizing variable v for the $\pi\pi$ scattering amplitude is depicted. The Roman numerals (I,..., IV) denote the images of the corresponding sheets; the thick line represents the physical region; the points i, 1 and $b = \sqrt{(m_K+m_\pi)/(m_K-m_\pi)}$ correspond to the $\pi\pi, K\overline{K}$ thresholds and $s=\infty$, respectively; the shaded intervals

$$(-\infty, -b], \qquad [-b^{-1},\ b^{-1}], \qquad [b, \infty)$$

[1] The analogous uniformizing variable has been used, *e.g.*, in ref. [23] in studying the forward elastic $p\bar{p}$ scattering amplitude.

are the images of the corresponding edges of the left-hand cut. The depicted positions of poles ($*$) and of zeros ($\circ$) give the representation of the type a) resonance in S_{11}.

On the v-plane, S_{11} has no cuts; however, S_{12} and S_{22} do have the cuts which arise from the left-hand cut on the s-plane, starting at $s = 4(m_K^2 - m_\pi^2)$, which further is neglected in the Riemann-surface structure, and the contribution of this cut is taken into account in the $K\overline{K}$ background as a pole on the real s-axis on the physical sheet in the sub-$K\overline{K}$-threshold region.

On v-plane, the Le Couteur-Newton relations are [13, 22]

$$S_{11} = \frac{d(-v^{-1})}{d(v)}, \quad S_{22} = \frac{d(v^{-1})}{d(v)}, \quad S_{11}S_{22} - S_{12}^2 = \frac{d(-v)}{d(v)}. \tag{4}$$

The $d(v)$-function already does not possess branch points and is taken as

$$d = d_B d_{\mathrm{res}}, \tag{5}$$

where $d_B = B_\pi B_K$; B_π contains the possible remaining $\pi\pi$ background contribution, related to exchanges in crossing channels (the consequent analysis gives $B_\pi = 1$); B_K is that part of the $K\overline{K}$ background which does not contribute to the $\pi\pi$ scattering amplitude:

$$B_K = v^{-4}(1 - v_0 v)^4 (1 + v_0^* v)^4. \tag{6}$$

The fourth power in (6) is stipulated by the following model-independent arguments [15]. First, a pole on the real s-axis on the physical sheet in S_{22} is accompanied by a pole in sheet II at the same s-value (as seen from eqs. (2)). On the v-plane this implies the pole of second order (and also zero of the same order, symmetric to the pole with respect to the real axis). Second, for the s-channel process $\pi\pi \to K\overline{K}$, the crossing u- and t-channels are the $\pi - K$ and $\overline{\pi} - K$ scattering (exchanges in these channels give contributions on the left-hand cut). This results in the additional doubling of the multiplicity of the indicated pole on the v-plane. So, the model of the $K\overline{K}$ background is determined by poles (here by the single one) on the real s-axis at the left-hand cut position on the physical sheet.

The function $d_{\mathrm{res}}(v)$ represents the contribution of resonances, described by one of three types of the pole-zero clusters, *i.e.*,

$$d_{\mathrm{res}} = v^{-M} \prod_{n=1}^{M} (1 - v_n^* v)(1 + v_n v), \tag{7}$$

where M is the number of pairs of the conjugate zeros.

3 Analysis of experimental data

We simultaneously analyze the available experimental data on the $\pi\pi$ scattering [25] and the process $\pi\pi \to K\overline{K}$ [26] in the channel with $I^G J^{PC} = 0^+0^{++}$. As data, we use the results of phase analyses which are given for phase shifts of the amplitudes (δ_1 and δ_{12}) and for moduli of the S-matrix elements $\eta_1 = |S_{aa}|$ ($a = 1\pi\pi$, $2K\overline{K}$)

Table 1. Demonstration of the quality of fits to the experimental data.

Variant	1				2			
Quantity	δ_1	η	δ_{12}	ξ	δ_1	η	δ_{12}	ξ
Number of experimental points	160	50	80	33	160	50	80	39
χ^2/N_{DF}	2.7	0.72	2.37	1.1	2.85	0.82	3.98	0.92
χ^2/N_{DF}	1.96		2.01		2.01		3	
χ^2/N_{DF}	1.98				2.45			
v_0 (s_0, GeV^2)	$0.954381 + 0.29859i$ (0.441)				$0.97925 + 0.202657i$ (0.6466)			
Variant	3				4			
Quantity	δ_1	η	δ_{12}	ξ	δ_1	η	δ_{12}	ξ
Number of experimental points	160	50	80	42	160	50	80	42
χ^2/N_{DF}	2.38	0.8	2.25	0.92	2.57	0.85	4.74	1.05
χ^2/N_{DF}	1.72		1.8		1.81		3.49	
χ^2/N_{DF}	1.76				2.59			
v_0 (s_0, GeV^2)	$0.954572 + 0.29798i$ (0.4646)				$0.982091 + 0.188405i$ (0.678)			

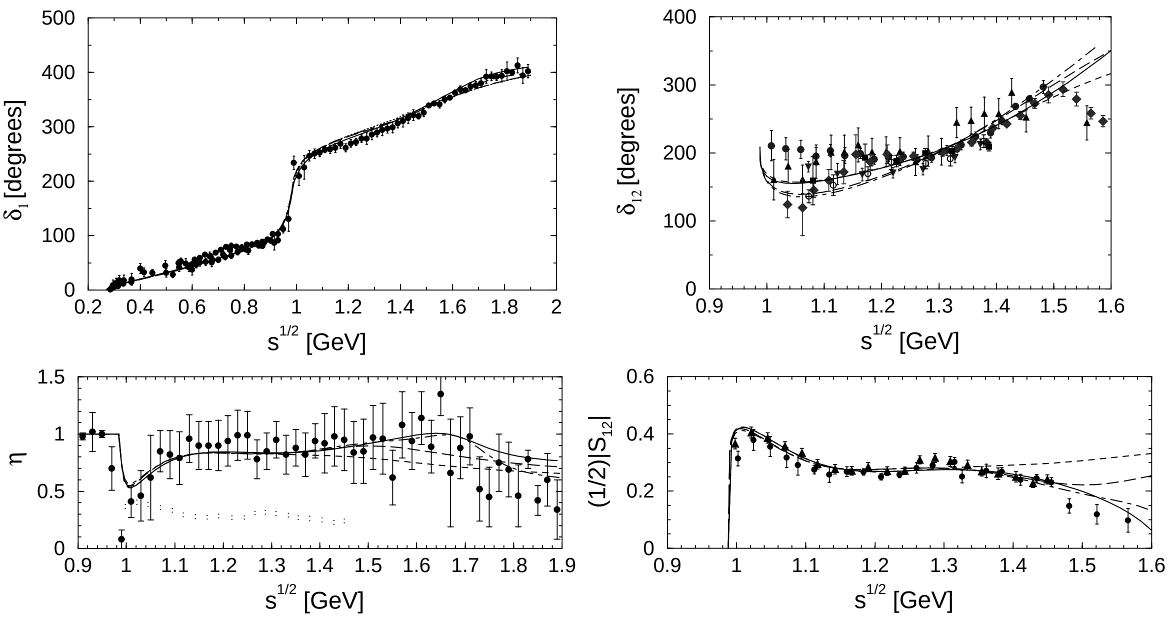

Fig. 2. Energy dependence of phase shifts and moduli of the matrix elements of processes $\pi\pi \to \pi\pi, K\overline{K}$ obtained on the basis of a combined analysis of the experimental data: the short-dashed lines correspond to variant 1; the long-dashed curves, to variant 2; the dot-dashed ones, to variant 4; and the solid lines, to variant 3. The data for the $\pi\pi$ scattering are taken from ref. [25]; for the process $\pi\pi \to K\overline{K}$, from ref. [26].

and $\xi = |S_{12}|$. The 2-channel unitarity condition gives $\eta_1 = \eta_2 = \eta, \xi = (1-\eta^2)^{1/2}, \quad \delta_{12} = \delta_1 + \delta_2$.

We consider four variants, in which the following states are taken into account:

Variant 1: The $f_0(665)$ and $f_0(980)$) with the clusters of type a), and $f_0(1500)$, of type c);

Variant 2: The same three resonances + the $f_0(1370)$ of type b);

Variant 3: The $f_0(665)$, $f_0(980)$) and $f_0(1500)$ + the $f_0(1710)$ of type b);

Variant 4: All the five resonances of the indicated types.

The other possibilities of the representation of these states are rejected by our analysis. We consider these variants, because the minimal possibility of the simultaneous description of two coupled processes $\pi\pi \to \pi\pi, K\overline{K}$ is realized without the $f_0(1370)$ and $f_0(1710)$, as is shown in our work [15]. The $\pi\pi$ scattering data are described from the threshold to 1.89 GeV in all the four variants (in addition, we take $B_\pi = 1$) and are taken from the analysis by B. Hyams *et al.* [25] in this energy region, and below

Table 2. Pole clusters for the considered resonances in variant 3.

Sheet		II	III		IV
$f_0(665)$	E, MeV	570 ± 13	700 ± 15		
	Γ, MeV	590 ± 24	72 ± 5		
$f_0(980)$	E, MeV	989 ± 5	982 ± 14		
	Γ, MeV	29 ± 7	195 ± 21		
$f_0(1500)$	E, MeV	1505 ± 23	1490 ± 30	1510 ± 25	1430 ± 20
	Γ, MeV	272 ± 25	220 ± 26	370 ± 30	275 ± 32
$f_0(1710)$	E, MeV		1680 ± 20		1700 ± 15
	Γ, MeV		114 ± 15		138 ± 21

1 GeV, from many works [25]. For the reaction $\pi\pi \to K\overline{K}$, practically all the accessible data are used, but the description ranges are slightly different for various variants and extend from the threshold to ~ 1.4 GeV for variant 1, to ~ 1.46 GeV for variant 2, and to ~ 1.5 GeV for variants 3 and 4. Table 1 demonstrates the quality of fits to the experimental data (the number of fitted parameters is 17 for variant 1, 21 for variants 2 and 3, 25 for variant 4). When calculating $\chi^2/N_{\rm DF}$, we have rejected the experimental points at 0.61, 0.65, and 0.73 GeV for δ_1, at 0.99, 1.65, and 1.85 GeV for η, at 1.111, 1.163, and 1.387 GeV for δ_{12}, and at 1.002, 1.265, and 1.287 GeV for ξ that give an especially large contribution to χ^2. We note that two variants (1 and 3) are the best, both without the $f_0(1370)$, and we stress that this analysis uses the *parameterless* description of the $\pi\pi$ background.

Let us indicate the obtained zero positions, on the v-plane, of the corresponding resonances:

<u>Variant 1</u>:

$$\begin{aligned}
\text{for} f_0(665): \quad & v_1 = 1.36964 + 0.208632i, \\
& v_2 = 0.921962 - 0.25348i, \\
\text{for} f_0(980): \quad & v_3 = 1.04834 + 0.0478652i, \\
& v_4 = 0.858452 - 0.0925771i, \\
\text{for} f_0(1500): \quad & v_5 = 1.2587 + 0.0398893i, \\
& v_6 = 1.2323 - 0.0323298i, \\
& v_7 = 0.809818 - 0.019354i, \\
& v_8 = 0.793914 - 0.0266319i,
\end{aligned}$$

<u>Variant 2</u>:

$$\begin{aligned}
\text{for} f_0(665): \quad & v_1 = 1.36783 + 0.212659i, \\
& v_2 = 0.921962 - 0.25348i, \\
\text{for} f_0(980): \quad & v_3 = 1.04462 + 0.0479703i, \\
& v_4 = 0.858452 - 0.0925771i, \\
\text{for} f_0(1370): \quad & v_5 = 1.22783 - 0.0483842i, \\
& v_6 = 0.802595 - 0.0379537i, \\
\text{for} f_0(1500): \quad & v_7 = 1.2587 + 0.0398893i, \\
& v_8 = 1.24837 - 0.0358916i, \\
& v_9 = 0.804333 - 0.0179899i, \\
& v_{10} = 0.795579 - 0.0253985i,
\end{aligned}$$

<u>Variant 3</u>:

$$\begin{aligned}
\text{for} f_0(665): \quad & v_1 = 1.38633 + 0.230588i, \\
& v_2 = 0.904085 - 0.263033i, \\
\text{for} f_0(980): \quad & v_3 = 1.05103 + 0.0487473i, \\
& v_4 = 0.864109 - 0.0922272i, \\
\text{for} f_0(1500): \quad & v_5 = 1.2477 + 0.0321349i, \\
& v_6 = 1.24027 - 0.0384191i, \\
& v_7 = 0.804333 - 0.0179899i, \\
& v_8 = 0.795579 - 0.0253985i, \\
\text{for} f_0(1710): \quad & v_9 = 1.25928 - 0.0115127i, \\
& v_{10} = 0.795429 - 0.00629969i.
\end{aligned}$$

<u>Variant 4</u>:

$$\begin{aligned}
\text{for} f_0(665): \quad & v_1 = 1.37103 + 0.21659i, \\
& v_2 = 0.917731 - 0.256026i, \\
\text{for} f_0(980): \quad & v_3 = 1.0457 + 0.0507053i, \\
& v_4 = 0.858452 - 0.0925771i, \\
\text{for} f_0(1370): \quad & v_5 = 1.22781 - 0.0496592i, \\
& v_6 = 0.801009 - 0.0420258i, \\
\text{for} f_0(1500): \quad & v_7 = 1.25817 + 0.0397054i, \\
& v_8 = 1.25078 - 0.0350769i, \\
& v_9 = 0.809333 - 0.0179899i, \\
& v_{10} = 0.795579 - 0.0253985i, \\
\text{for} f_0(1710): \quad & v_{11} = 1.2604 - 0.00927258i, \\
& v_{12} = 0.794694 - 0.00458088i.
\end{aligned}$$

Figures 2 demonstrate the comparison of the obtained energy dependence of the four analyzed quantities with the experimental data: The short-dashed lines correspond to variant 1; the long-dashed curves, to variant 2; the dot-dashed ones, to variant 4; and the solid lines, to variant 3.

In tables 2 and 3, the obtained pole clusters of the considered resonances are shown on the corresponding sheets on the complex energy plane ($\sqrt{s_r} = E_r - i\Gamma_r$) for the best variant 3 (without the $f(1370)$) and for variant 4 (with all five states).

The coupling constants of the obtained states with $\pi\pi$ (g_1) and $K\overline{K}$ (g_2) systems are calculated through the residues of amplitudes at the pole on sheet II for resonances of types a) and c), and on sheet IV for resonances

Table 3. Pole clusters for the considered resonances in variant 4.

Sheet		II	III		IV
$f_0(665)$	E, MeV	600 ± 16	715 ± 17		
	Γ, MeV	605 ± 28	59 ± 6		
$f_0(980)$	E, MeV	985 ± 5	984 ± 18		
	Γ, MeV	27 ± 8	210 ± 22		
$f_0(1370)$	E, MeV		1310 ± 22		1320 ± 20
	Γ, MeV		410 ± 29		275 ± 25
$f_0(1500)$	E, MeV	1528 ± 22	1490 ± 30	1510 ± 20	1510 ± 21
	Γ, MeV	385 ± 25	220 ± 24	370 ± 30	308 ± 30
$f_0(1710)$	E, MeV		1700 ± 25		1700 ± 20
	Γ, MeV		86 ± 16		115 ± 20

Table 4. Coupling constants of obtained states with $\pi\pi$ (g_1) and $K\overline{K}$ (g_2) systems.

	$f_0(665)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$
g_1, GeV	0.652 ± 0.065	0.167 ± 0.05	0.116 ± 0.03	0.657 ± 0.113
g_2, GeV	0.724 ± 0.1	0.445 ± 0.031	0.99 ± 0.05	0.666 ± 0.15

of type b). Expressing the T-matrix via the S-matrix as

$$S_{ii} = 1 + 2i\rho_i T_{ii}, \qquad S_{12} = 2i\sqrt{\rho_1\rho_2}T_{12}, \tag{8}$$

where $\rho_i = \sqrt{(s-4m_i^2)/s}$, and taking the resonance part of the amplitude as

$$T_{ij}^{\rm res} = \sum_r g_{ir}g_{rj}D_r^{-1}(s) \tag{9}$$

with $D_r(s)$ being an inverse propagator ($D_r(s) \propto s - s_r$), we show the results of that calculation in table 4. We see that the $f_0(980)$ and especially the $f_0(1370)$ are coupled essentially more strongly to the $K\overline{K}$ system than to the $\pi\pi$ one, which tells about the dominant $s\bar{s}$ component in these states. The $f_0(1500)$ has the approximately equal coupling constants with the $\pi\pi$ and $K\overline{K}$ systems, which apparently could point up to its dominant glueball component [27]. The coupling constant of the $f_0(1710)$ with the $\pi\pi$-channel cannot be calculated by this method, unless the description of $\pi\pi \to K\overline{K}$ reaction is obtained in the region of this resonance. But this state is represented by the cluster corresponding to the dominant $s\bar{s}$ component.

Let us also present the calculated scattering lengths. For the $K\overline{K}$ scattering:

$$a_0^0 = -1.25 \pm 0.11 + (0.65 \pm 0.09)i,\ [m_{\pi^+}^{-1}];\ \text{(variant1)},$$
$$a_0^0 = -1.548 \pm 0.13 + (0.634 \pm 0.1)i,\ [m_{\pi^+}^{-1}];\ \text{(variant2)},$$
$$a_0^0 = -1.19 \pm 0.08 + (0.622 \pm 0.07)i,\ [m_{\pi^+}^{-1}];\ \text{(variant3)},$$
$$a_0^0 = -1.58 \pm 0.12 + (0.59 \pm 0.1)i,\ [m_{\pi^+}^{-1}];\ \text{(variant4)}.$$

The presence of the imaginary part in $a_0^0(K\overline{K})$ reflects the fact that already at the threshold of the $K\overline{K}$, other channels ($2\pi, 4\pi$, etc.) are opened. Variants 2 and 4 include the $f_0(1370)$ unlike variants 1 and 3. We see that Re $a_0^0(K\overline{K})$ is very sensitive to whether this state exists or not.

In table 5, we compare our results for the $\pi\pi$ scattering length a_0^0 with results of some other theoretical and experimental works.

At first, let us remark about the result of ref. [18] for the $\pi\pi$ scattering length. We think that such a small value ($0.23\ m_{\pi^+}^{-1}$) has been obtained, because there has been used an assumption about the negative $\pi\pi$ background in the phase shift. Now, from table 5 we see that our results correspond to the linear realization of chiral symmetry.

We have here presented model-independent results: the pole positions, coupling constants and scattering lengths. Masses and widths of these states that should be calculated from the obtained pole positions and coupling constants are highly model dependent. For instance, if we suppose that the $f_0(665)$ is the σ-meson, then from the known relation

$$g_{\sigma\pi\pi} = \frac{m_\sigma^2 - m_\pi^2}{\sqrt{2}f_{\pi^0}}$$

(here f_{π^0} is the constant of weak decay of the π^0: $f_{\pi^0} = 93.1$ MeV), we obtain $m_\sigma \approx 342$ MeV.

If we take the resonance part of amplitude in a non-relativistic form (see [1], p. 214)

$$T^{\rm res} = \frac{\Gamma_{\rm el}/2}{E_\sigma - E - i\Gamma_{\rm tot}/2},$$

then we have $E_\sigma \approx 570 \pm 21$ MeV and $\Gamma_{\rm tot} \approx 1400 \pm 30$ MeV; for the so-called relativistic form

$$T^{\rm res} = \frac{\sqrt{s}\Gamma_{\rm el}}{m_\sigma^2 - s - i\sqrt{s}\Gamma_{\rm tot}},$$

the following values are obtained: $m_\sigma \approx 850 \pm 20$ MeV and $\Gamma \approx 1240 \pm 30$ MeV.

Table 5. Comparison of theoretical and experimental values for the $\pi\pi$ scattering length a_0^0.

$a_0^0,\ m_{\pi^+}^{-1}$	References	Remarks
0.27 ± 0.06 (1) 0.267 ± 0.07 (2) 0.28 ± 0.05 (3) 0.27 ± 0.08 (4)	Our paper	Model-independent approach
0.26 ± 0.05	L. Rosselet *et al.* [25]	Analysis of the decay $K \to \pi\pi e\nu$ using Roy's model
0.24 ± 0.09	A.A. Bel'kov *et al.* [25]	Analysis of $\pi^- p \to \pi^+\pi^- n$ using the effective range formula
0.23	S. Ishida *et al.* [18]	Modified analysis of $\pi\pi$ scattering using Breit-Wigner forms
0.16	S. Weinberg [28]	Current algebra (non-linear σ-model)
0.20	J. Gasser, H. Leutwyler [29]	One-loop corrections, non-linear realization of chiral symmetry
0.217	J. Bijnens *et al.* [30]	Two-loop corrections, non-linear realization of chiral symmetry
0.26	M.K. Volkov [31]	Linear realization of chiral symmetry
0.28	A.N. Ivanov, N. Troitskaya [32]	A variant of chiral theory with linear realization of chiral symmetry

4 Conclusions

On the basis of a simultaneous description of the isoscalar s-wave channel of the processes $\pi\pi \to \pi\pi, K\overline{K}$ with a parameterless representation of the $\pi\pi$ backgound, a model-independent confirmation of the σ-meson below 1 GeV is obtained. We emphasize that this is a real evidence of this state, because we have not been enforced to construct the $\pi\pi$ background.

A parameterless description of the $\pi\pi$ background is given only by allowance for the left-hand branch point in the proper uniformizing variable. This seems to be related to the fact that the exchanges by nearest σ- and ρ-mesons in the crossing channels contribute to the $\pi\pi$ scattering amplitude with opposite signs (due to gauge invariance) and compensate each other.

Note also that a light σ-meson is needed, for example, for an explanation of $K \to \pi\pi$ transitions using the Dyson-Schwinger model [33] and for an explanation of the experimental value of the pion-nucleon Σ-term ($\Sigma_{\pi N} \sim$ 40–70 MeV) in a linear σ-model based on the $U(3) \times U(3)$ quark effective Lagrangian [34].

Since all the fitted parameters in describing the $\pi\pi$ scattering are only the positions of poles corresponding to resonances, we conclude that our model-independent approach is a valuable tool for studying the realization schemes of chiral symmetry. The existence of the low-lying state $f_0(665)$ with the properties of the σ-meson and the obtained $\pi\pi$ scattering length ($a_0^0(\pi\pi) \approx 0.27[m_{\pi^+}^{-1}]$) suggest the linear realization of chiral symmetry.

The analysis of the used experimental data gives evidence that the $f_0(980)$- and especially the $f_0(1370)$-resonance (if it exists —variants 2 and 4), have the dominant $s\bar{s}$ component. Note that a minimum scenario of the simultaneous description of processes $\pi\pi \to \pi\pi, K\overline{K}$ goes without the $f_0(1370)$-resonance. The best total $\chi^2/N_{\rm DF}$ for both the analyzed processes is obtained with the set of states: $f_0(665)$, $f_0(980)$, $f_0(1500)$ and $f_0(1710)$. The $K\overline{K}$ scattering length is very sensitive to whether the $f_0(1370)$ state exists or not.

The $f_0(1500)$ has the approximately equal coupling constants with the $\pi\pi$ and $K\overline{K}$ systems, which apparently could point up to its dominant flavour-singlet (*e.g.*, glueball) component [27].

The $f_0(1710)$ is represented by the cluster corresponding to the state with the dominant $s\bar{s}$ component. Although the lattice simulations suggest that the lowest mass state of a pure glue would be the 0^{++} with a mass of 1670 ± 20 MeV [35], our result is in accord with refs. [8, 27], where the $f_0(1500)$ has been considered as a candidate for the scalar glueball. Note that QCD sum rules [36] and the K-matrix method [3] showed both the $f_0(1500)$ and $f_0(1710)$ to be mixed states with large admixture of the glueball component. Their conclusions about the glueball component is in agreement with our conclusion as to the $f_0(1500)$ but not to $f_0(1710)$. It seems that the complement of the our combined consideration by the $\eta\eta$- and $\eta\eta'$-channels should not change substantially this situation in view of the relative distance of the $\eta\eta$ threshold, and because the glueball component is not coupled to the $\eta\eta'$ system. Note also that the conclusion of QCD sum rules [36] about the existence of light glueballs (below 1 GeV) contradicts the lattice calculations and is not confirmed by our method.

We stress that our results are very decisive, because our approach is based only on the first principles (analyticity-microcausality and unitarity), immediately applied to the analysis of experimental data, and it is free from dynamical assumptions, because a way of its realization is based on the *mathematical* fact that a local behaviour of analytic functions determined on the Riemann surface is governed by the nearest singularities on all corresponding sheets. It is very important that we were able to describe the considered coupled processes without diminishing the number of fitted parameters by some dynamical assumptions.

We think that multichannel states are most adequately represented by clusters, *i.e.*, by the poles on all the corresponding sheets. Pole clusters give a main effect of resonances, and on the uniformization plane they are their good representation. The pole positions are rather stable characteristics for various models, whereas masses and widths are very model dependent for wide resonances. Earlier one noted that the wide-resonance parameters are largely controlled by the non-resonant background (see, *e.g.*, [37]). In part this problem is removed by the parameterless and natural description of the $\pi\pi$ background; there remains only a considerable dependence of resonance masses and widths on the used model. Therefore, for those states it makes little sense to publish masses and widths. It seems to be more right to publish the pole positions on all corresponding sheets. To specify a pole cluster, we propose to use its centre on the complex-energy plane (the real part of this centre). We have made this for the σ-meson (variant 1) owing to its large definition in the Particle Data Group tables.

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